Reg. No. : [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Question Paper Code : 42772

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fifth Semester

Computer Science and Engineering

MA2265 – DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulations 2008)

(Common to PTMA2265 – Discrete Mathematics for B.E. (Part-Time)

Third Semester – CSE – Regulations 2009)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Use Truth table, check whether $(P \wedge Q) \vee (\neg P \vee \neg Q)$ is a tautology or contradiction.

2. State the truth value of the statement : 'If tiger has wings, then the earth travel round the sun'.

3. State and prove Pigeonhole principle.

4. How many positive integers n can be formed using the digits 3, 4, 5, 5, 6, 6, 7 if n has to exceed 50,00,000 ?

5. Check whether the graph $K_{2,4}$ is Eulerian or Hamiltonian. Justify the claim.

6. What is meant by mixed graph ?

7. Let Z denote the set of all integers. A binary operation * is defined on Z by $a * b = a + b - ab$ for all a, b in Z. Is (Z, *) a semigroup ?

8. Give an example of a ring which is not a field . Justify the claim.

9. Show that every totally ordered set is a lattice.

10. Give an example of a lattice which is complemented but not distributive.

PART – B (5×16=80 Marks)

11. a) i) Use truth table to show that $(P \to R) \wedge (Q \to R) \equiv (P \vee Q) \to R$. (8)

ii) Give a proof by contradiction of the theorem "If $3n + 2$ is odd, then show that n is odd. (8)

(OR)

b) i) Show that $A \wedge S$ can be derived from the premises $P \to Q$, $Q \to \neg R$, R, $P \vee (A \wedge S)$. (8)

ii) Show that $\neg(\forall x)\ (P(x) \to Q(x))$ and $(\exists x)\ (P(x) \wedge \neg Q(x))$ are logically equivalent. (8)

12. a) i) Prove that if m is an odd positive integer, then there exists a positive integer n such that m divides $2^n - 1$. (6)

ii) Solve $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$, $n \geq 2$, $a_0 = 0$, $a_1 = 7$. (10)

(OR)

b) i) Find the number n, $1 \leq n \leq 1000$ such that n is not divisible by 2, 3 or 5. (8)

ii) Solve $a_n - 2a_{n-1} - 3a_{n-2} = 0$, $n \geq 2$, $a_0 = 3$, $a_1 = 1$. (8)

13. a) i) Show that an undirected graph has an even number of vertices of odd degree. (8)

ii) If G is a simple graph with n vertices and k-components, then show that the number of edges is at most $(n - k)(n - k + 1)/2$. (8)

(OR)

b) i) Test whether the graphs with the following adjacency matrices

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

are isomorphic or not. (8)

ii) Show that the complete bipartite graph $K_{m,n}$ is Hamiltonian if and only if $m = n$. (8)

14. a) i) If every element in a group is its own inverse, then show that G is an abelian group. (4)

ii) Show that the order of a subgroup H of a finite group G divides the order of the group G. (12)

(OR)

b) i) Show that the set of all permutations of three distinct elements with right composition of permutation is a permutation group. (10)

ii) Show that if $f : \langle G, * \rangle \rightarrow \langle H, \Delta \rangle$ is a group homomorphism, then Ker(f) is a normal subgroup of the group G. (6)

15. a) i) Let $(P, \leq)$ be a poset. If the least element and greatest element exist, then show that they are unique. (6)

ii) Show that in a lattice, isotone property and distributive inequalities are true. (10)

(OR)

b) Show that in a complemented and distributive lattice L, the following are true. For all x, y in L,

i) $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. (10)

ii) $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. (6)
